Plan for today

1. Number systems
2. Binary, hex, base $n$
3. Negative integers
4. Floating point
Number systems

![Binary Sudoku puzzle](https://xkcd.com/826/)

**v. 08/23/2013**

SPIS 2013 Foundations of CS  
Lecture August 20th, 2013
How do you compute the number represented as

111

if we’re using

- Decimal representation of numbers?
- Binary representation of numbers?
- Ternary representation of numbers?
- Hexadecimal representation of numbers?
- Base $n$ representation of numbers?
Example

How do you compute the number represented as

111

if we’re using

- Decimal representation of numbers? base 10
- Binary representation of numbers? base 2
- Ternary representation of numbers? base 3
- Hexadecimal representation of numbers? base 16
- Base \( n \) representation of numbers?
Why use bases other than 10?

- Binary
- Hexadecimal $2^4 = 16$
What is decimal value of

1001010100b

?
Converting from (unsigned) binary to decimal

What is decimal value of

\[ \begin{array}{cccccccccc}
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & b \\
2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^0 \\
\end{array} \]

Multiply each bit by appropriate power of 2
Converting from (unsigned) binary to decimal

What is decimal value of

\[
\begin{array}{cccccccc}
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
2^9 & 2^8 & 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 \end{array}
\]

Multiply each bit by appropriate power of 2

\[512 + 64 + 16 + 4 = 596d.\]
Converting from decimal to (unsigned) binary

What is binary representation of 17\textit{d}?

A. 17\textit{b}  
B. 10\textit{b}  
C. 101\textit{b}  
D. 1001\textit{b}  
E. 10001\textit{b}
Converting from decimal to (unsigned) binary

What is binary representation of $17d$?

A. $17b$  
B. $10b$  
C. $101b$  
D. $1001b$  
*E. $10001b$
Part 2 of this week’s lab is all about **number conversions**

- **numToBinary**
  - Argument: number to be converted
  - Returns: string binary representation

  *Easiest* conversion algorithm is right-to-left and uses **recursion**.

If you want to implement the harder algorithm too, try to do the conversion left-to-right.

- **Bonus: binaryToNum**
  - Argument: string binary representation
  - Returns: decimal value

  Disclaimer: requires more string manipulations.
Hexadecimal

Number systems pros and cons:

- **Decimal**
  - **Pros** familiar, relatively concise  
  - **Cons** Hard to translate to signals
- **Binary**
  - **Pros** compatible with logic, circuits  
  - **Cons** Verbose

For example, the number $1000$ uses $4$ digits but would require $9$ bits:

$$1000_d = 1111101000_b$$

The hexadecimal system is base $16$.

Notation: number in hex starts with $0x$.

Digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$.

Since $16 = 2^4$, one hex digit replaces $4$ binary digits.
Hexadecimal

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- **Decimal**
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- **Notation**: number in hex starts with 0x.
- **Digits**: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Since 16 = 2^4, one hex digit replaces 4 binary digits.
## Number systems

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Hex</th>
<th>Binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>110</td>
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<td>7</td>
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<td>11</td>
<td>B</td>
<td>1011</td>
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<tr>
<td>12</td>
<td>C</td>
<td>1100</td>
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<tr>
<td>13</td>
<td>D</td>
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</tr>
<tr>
<td>14</td>
<td>E</td>
<td>1110</td>
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<td>F</td>
<td>1111</td>
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<td>12</td>
<td>10010</td>
</tr>
<tr>
<td>19</td>
<td>13</td>
<td>10011</td>
</tr>
</tbody>
</table>
What is the binary representation of

0x5ABF1?

A. 101101010111111100010
B. 01011010101111110001
C. 00010010101001010111
D. 10101110101011101001
E. None of the above
Hexadecimal example

What is the binary representation of

\[ 0x5ABF1 \]?

A. 10110101011111100010
B. 01011010101111110001
C. 00010010101001010111
D. 10101110101011101001
E. None of the above
Hexadecimal numbers and colors
Flashback to Monday’s lecture

“1 byte = 8 bits;
which can represent numbers from 0 to 255 (255 = 2^8 − 1)”

**Theorem:** An $n$ bit (unsigned) number can have value at most $2^n − 1$.

**Proof:**
Flashback to Monday’s lecture

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- What \(n\) bit number has the largest value?
- What value is it?
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  **Hint:** use sum notation
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- What value is it? **Hint: use sum notation**

*Induction to the rescue!!!*
There are only 10 types of people in the world: Those who understand binary and those who don't.
Non-integers?

How do you compute the number represented as

\[ 10.111 \]

if we’re using

- Decimal representation of numbers?
- Binary representation of numbers?
Non-integers?

How do you compute the number represented as

10.111

if we’re using

- Decimal representation of numbers?  base 10
- Binary representation of numbers?  base 2
Non-integers?

How do you compute the number represented as

$$10.111$$

if we’re using

- Decimal representation of numbers? base 10
- Binary representation of numbers? base 2

Vote on decimal value if interpret as binary string:

A. 10.111
B. 23
C. $$2 \frac{7}{8}$$
D. $$\frac{23}{32}$$
Dealing with fractional parts of numbers:

- **Single precision**
  - 32 bits (4 bytes)
  - Significand has a precision of 24 bits (about 7 decimal digits)

- **Double precision**
  - 64 bits (8 bytes)
  - Significand has a precision of 53 bits (about 16 decimal digits)

- **Double extended**
  - At least 79 bits (80 if the hidden/implicit bit rule is not used)
  - Significand has a precision of at least 64 bits (about 19 decimal digits)
Balanced Ternary notation

A wacky number system you’ll explore in lab.

- Place values are same as ternary
  \[ \cdots 3^3 3^2 3^1 3^0 \]
- Symbols are
  \[ + ~ 0 ~ - \]

What is decimal value of the number represented by

\[ + -- \]

A. 0  
B. 5  
C. 6  
D. 42
End of math content for today
We’re at the halfway mark!!!

Program goal 1: Students will develop skills and attitudes that will lead to success in academic settings as well as in life.
We’re at the halfway mark!!!

- Take concrete steps toward solving their own problems, including:
  - Articulating the scope and details of a problem they face
  - Identifying one or more steps to make progress towards a solution
  - Trying multiple approaches if an original attempt fails
  - Looking up how to do something they didn’t know how to do
  - Identifying the appropriate resource to obtain help
  - Seeking help from this resource

- Start assignments early
- Go beyond the basic requirements of an assignment
- Work with a partner to solve a problem
- Employ one or more strategies for coping with frustration
- Identify their own strengths and areas for improvement (including knowledge they possess and knowledge they are missing)
- Identify as someone who can master material through hard work
- Approach complex problems and texts with confidence
- Engage instructors and staff in productive dialogue
- Connect academic activities to longer-term goals
Decorate the boards!

What’s working well?
- In your pair-programming partners
- In your writing workshop groups
  ... Any strategies or tricks you’ve come up with you can share?

What’s not working so well?
- In your pair-programming partners
- In your writing workshop groups
  ... Any specific stumbling blocks we can debug?
Partnerships in the wild

Google founders Larry Page and Sergey Brin

Apple founders Steve Jobs, Ronald Wayne, and Steve Wozniak

RSA inventors Ron Rivest, Adi Shamir, and Leonard Adleman
Who wants teamwork and communication?


Software Development Engineers dive deep into code and work as part of a team of programming experts who solve problems and build powerful new tools. You collaborate with Software Development Engineers in Test (SDETs) to ensure stellar product quality, and you provide technical guidance to Program Managers (PMs) as they communicate user needs and product requirements. As a SDE, you dedicate yourself to producing the world's most advanced software.
First steps

1. SPIS 😊
2. UCSD, e.g.